Rope System Analysis

By Stephen W. Attaway

Abstract

This paper presents an analysis of the loads in a typical climbing rope system subjected to a dynamic loading from a fall. Several examples are illustrated to show how to calculate the force on ropes and anchors subjected to dynamic loads that are experienced by a falling rock climber. The force in a rope that is generated when a falling weight is arrested depends on how fast the weight is stopped. We will use the energy method to solve for the maximum strain energy in the rope. The effects of friction, dynamic rope modulus, and rope condition will also be considered.

We developed some rules of thumb to help a lead climber place fall protection and understand the limitations. The amount of 'safe' lead out depended on the amount of rope that is between the belayer and the climber, the type and condition of the belay rope, and the type of anchor used.

Motivation

Rock Climbing is a technical sport. A good understanding of the mechanics of anchor placement, rope behavior, and impact dynamics is important to climber safety.

On June 23, 1996, three climbers fell to their death on the Warpy Moople route on the formation called Muralla Grande located in the Sandia Mountains east of Albuquerque, NM. Warpy Moople is a Grade III, class 5.9 climb with 8 pitches. At least one of the climbers had reached the top of the last pitch, which is rated 5.5-5.6. One plausible scenario is that the first climber reached the top and called "off belay" before placing his top rope belay anchor. The other two climbers may have begun removing their belay anchor and were getting ready to climb when the first climber fell. Only three pieces of protection were found on the 165 rope between the first climber and the next. One question that has been asked is: why did the protection fail. We know that at least 100 ft. of rope was between the lead climber and the belayer. We do not know the location of the protection. Two pieces of protection were of the cam type design, and the third was a chock with a wire. These three experienced climbers fell to their deaths believing that the cam type protection device could protect a fall of over 100ft without pulling out. Many other experienced rock climbers that I interviewed also believed that 50 to 100 ft. lead outs were 'safe'.

Here I will try to show that the amount of 'safe' lead out depends on the amount of rope that is between the belayer and the climber, the type and condition of the belay rope, and the type of anchor used. In the rest of this paper we will outline some of the methods of rock climbing and fall protection. Then, some equations that are useful for understanding rope behavior are derived. These equations will be applied to a typical leader fall to predict the magnitude of forces that an anchor must withstand. The effects of dynamic stiffness, friction, rope condition, and belay device will be considered. Finally, we will consider how much "lead out" is safe for a given rope and anchor system. Several areas for research on dynamic rope behavior are suggested.

Methods of rock climbing fall protection

Some of the typical styles of rock climbing are shown in Figure 1. Top-roping has a belayer at the top of the climb with a near taut rope going down to the climber. If the climber falls, the rope is weighted quickly by the climbers weight.

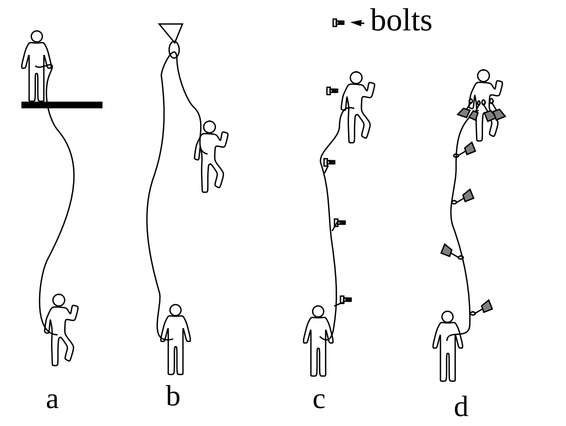


Figure 1. Styles of rock climbing. a) Top roping, b) Gym Climbing, c) Sport Climbing, d) Free Climbing.

Gym climbing usually has the belayer on the ground with the rope running up to a carabiner or pulley along the top of the climb. Here, if the climber falls, the rope is also quickly weighted by climbers weight, and the climber can be lowered to safety.

Sport climbing involves climbing a wall that has fixed (permanent) points of protection along its path. Typically rock bolts are placed at short intervals. The climber will start from the ground with a rope being lead out by the belayer. The climber can clip the rope into each fixed protection point using a sling with carabiners. Since the climber may be quite a distance above their protection, a climber fall can lead to high impact forces in the rope and anchor systems.

Free climbing is similar to sport climbing except that there are no fixed points of protection. Instead, the climber must wedge 'chocks' or 'cam' devices into the rock at set intervals. Before the lead climber reaches the end of the rope, a belay station is rigged so that the second climber can be belayed from the top in a top-rope style. The second climber will typically remove the fall protection as they climb. Free climbing requires expert skill in placing the fall protection. If placed incorrectly, the dynamic force from a fall can rip all the fall protection out and lead to a 'grounding'.

Rope Deflection

Before looking at the complex rope system, let's look at the dynamics of a simpler mass-spring model. Mass-spring systems are well studied, and usually engineers and physics majors are tortured by the equations for this system in college exams. A fallen climber on rope behaves somewhat like a mass-spring system. The spring corresponds to the rope, and the mass corresponds to the weight of the fallen climber. An ideal mass-spring system is shown in Figure 2

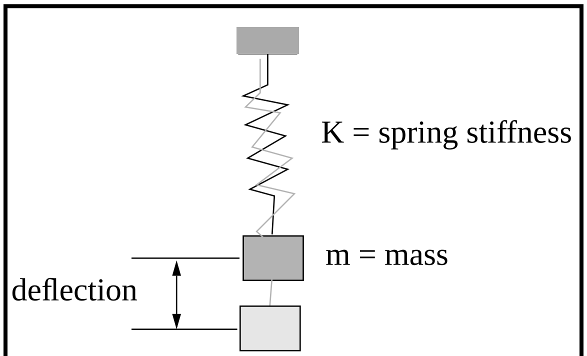


Figure 2. Simple mass-spring system.

If you were to follow a single strand of nylon in a rope, then you would find that it makes a spiral path along the length of the rope. This spiral path looks like the same path that a steel spring makes. This spring-like path is what gives a rope its 'bounce'. As the rope is loaded, the spring is stretched exactly as a steel spring is stretched.

Dynamic ropes are designed to stretch. For example, Gold Line climbing rope has nearly ten times more stretch than that of static PMI. Rock climbing ropes that are designed to take falls are also designed to stretch. Both static and dynamic ropes are made from the same type of nylon (Kevlar and Spectra have too little stretch to absorb energy). If you measured the stretch of a single strand of nylon, then you would find that it is much stiffer than the rope. The rope construction (number of core strands, twist of core strands, sheath tightness, size, weave of the sheath yarn, etc.) determines the modulus of a rope. (H. C. Wu , 1992, has developed an accurate method to predict the static tensile strength of double-braided ropes base on the above properties.) Note that most webbing does not have a spiral, and, thus, has very little stretch. This makes webbing a poor energy absorber.

Static Deflection

Before we can compute the dynamic forces from a fall, we must first understand how a rope responds to a static load. The force in the rope (spring) is proportional to the amount of stretch in a rope. The relation is

$$F = K\delta \qquad \text{Eq(1)}$$

where F is the force in the rope, K is called the stiffness of the rope, and δ is the displacement of the rope from its unstretched length.

The stiffness of the rope, K, depends on the length of the rope. Short ropes are very stiff, and long ropes are less stiff. The stiffness can be computed as:

$$K = \frac{M}{L} \qquad \text{Eq(2)}$$

where M is the called the rope modulus (change in force for a given stretch), and L is the rope length.[1] The rope modulus is computed from the stretch in a rope under a given load. The modulus is defined as the force per unit stretch:

$$M = \frac{F}{\varepsilon}, \qquad \text{Eq(3)}$$

where $\varepsilon = \frac{\delta}{L}$ is the stretch or the change in length over the length. The stiffness of a rope may change with load or as the rope is used.

Table 1 shows the typical rope moduli for different types of ropes. Notice that the modulus of static rope is four to five time stiffer than dynamic rope.

As an example of using the equations of a spring, consider a weight of W= 200 lbs on PMI rope. At L=200 ft., the static deflection should be given by:

$$\delta = \frac{W}{K} = \frac{WL}{M} = \frac{200 \times 200}{19555} = 2.0\,ft. \qquad \text{Eq(4)}$$

If two 200 lb loads were on the rope at the same time, then the stretch would be 4 ft. If the rope were twice as long, say 400 ft., then the 200 lb load would stretch the rope 4 ft., an 800 ft. rope would stretch 8 ft. If Gold Line were used, on an 800 ft. rope, 80 ft. of stretch would be required to support a 200 lb load.

1. A note to engineers: This is not a conventional modulus. It has dimensions of force, not stress.

Table 1: Static modulus of different types of rope.

	Force	stretch (δ/L)	Modulus (lb/ft/ft)
PMI [1]	176 lbs	0.009	19555
Blue Water II	176 lbs	0.011	16000
Gold Line	176 lbs	0.088	2000
dynamic climbing rope [2]			4000 - 8000

One way to determine the modulus of a rope is to measure the deflection for a given weight on 100 ft. of rope. The static modulus can then be computed based on:

$$M = \frac{\overline{W}\overline{L}}{\bar{\delta}} \quad \text{Eq(5)}$$

where the bar above the quantities indicates measured results. Here we have used the term static modulus to indicate that the modulus was measured under a static (non-moving) loading condition. Later, we will introduce the concept of a dynamic modulus.

The modulus of a dynamic belay rope will change with use. A fall on a dynamic rope will straighten some of the fibers and cause the rope to become stiffer. The rope modulus can also change if it is used for climbing or rappelling.

Dynamic loads

Rope stretch is important because it governs the distance over which a falling load will stop. The shorter the stopping distance, the greater the deceleration. Since the dynamic force in the rope is equal to the mass times the deceleration, high decelerations mean high loads in the rope. (the fall does not hurt; its that sudden stop at the end).

One way of approximating the maximum dynamic force in a rope system is to use an energy balance equation. (see Spotts, 1978 [4]) The total energy of a fall must be balanced by the total strain energy in the rope. At the end of the fall, the rope will be stretched to its maximum length. At this point, the climber will have just come to zero velocity, and all the energy from the fall will be stored in the rope as strain energy. To compute the maximum load in the rope during a fall, we will need to know the maximum stretch in the rope.

All the energy added to a falling weight as it travels through the earth's gravity field must be converted into strain energy in the rope. The energy balance for the mass-spring system is:

$$PE = SE \quad \text{Eq(6)}$$

where PE is the potential energy, and SE is the strain energy in the rope.

Potential Energy

For a mass moving through the earth's gravity, the change in potential energy is given by:

$$PE = mgh \quad \text{Eq(7)}$$

where h is the height or distance of the fall, g is the gravity constant, and m is the mass. We have neglected air resistance here. One can argue that air drag is proportional to velocity to a good approximation and that for velocities encountered in 'safe' falls air drag is negligible. If you were going to bungee jump from the top if El Cap, then you may want to consider air drag.

If a weight falls a given height, *h*, the potential energy is converted to kinetic energy according to:

$$\frac{1}{2}mv^2 = mgh \quad \text{Eq(8)}$$

where m is the mass, and v is the velocity. To stop a fall, a rope must absorb all of the kinetic energy, which it does by converting it into strain energy.

We measure the height, *h*, in the potential energy equation from the lead climbers location down to the point where the rope has no stretch. Because the mass will move downward as the spring stretches, we must also include the additional change in potential energy due to the stretch of the rope. The total change in potential energy due to the fall from height, h, and the deflection of the rope, δ, will be:

$$PE = mgh + mg\delta \quad \text{Eq(9)}$$

Strain Energy

Now, lets look at the energy used in stretching a spring. The strain energy (or work done) of the spring is given by the force in the spring integrated over the distance which it acts. Because the force changes as the rope is stretched, the strain energy is computed by integrating the force over the displacement of the spring.

$$SE = \int_0^{\delta} F(x)dx \quad \text{Eq(10)}$$

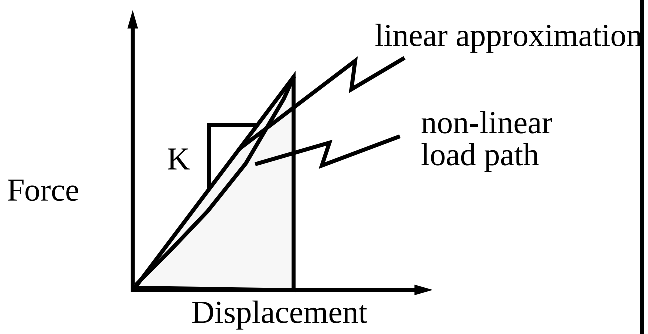


Figure 3. Force - Displacement relation for a spring. The strain energy is the area under the curve. The stiffness, K, corresponds to the slope of the force-displacement curve.

If the spring is linear (meaning it has the same stiffness for a given displacement of interest) then the strain energy is:

$$SE = \frac{1}{2}K\delta^2 \qquad \text{Eq(11)}$$

where K is the stiffness in the spring (rope), and δ is the maximum displacement of the spring (stretch in the rope). Note that some ropes, like bungee cord, may not have a linear force displacement curve. The energy method can still be used; however, the math will become more difficult because of the integral in Eq(10). Often, a rope will be 'almost' linear. In this case, we approximate the stiffness of the rope as shown in Figure 3. For now, we will assume linear force displacement relations as a first approximation.

Now that we have defined the relations for strain energy, kinetic energy, and potential energy, we can use these relations to solve for the maximum force generated by a fall.

$$Wh + W\delta - \frac{1}{2}K\delta^2 = 0 \qquad \text{Eq(12)}$$

where W is the weight of the climber. Recall that W = mg. We can now use the quadratic equation to solve for the displacement:

$$\delta = \frac{W \pm \sqrt{W^2 + 2KWh}}{K} \qquad \text{Eq(13)}$$

This simplifies to,

$$K\delta = W + W\sqrt{1 + 2\frac{K}{W}h} \qquad \text{Eq(14)}$$

We can express Eq(13) in terms of the static deflection of the rope under the weight, W, by using Eq(4) $\frac{W}{K} = \delta_{st}$. Recalling from Eq(1) that $F = K\delta$, and assuming K is the same under dynamic and static loading, allow us to express the ratio of the dynamic force in the rope to the climbers weight in terms of the initial fall height and the static rope deflection. From the solution to the quadratic equation and substitution we get:

$$\frac{F}{W} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \qquad \text{Eq(15)}$$

The ratio F/W is called the **impact load factor**. Another way to look at the impact load factor is that F/W corresponds to the maximum number of g's that the climber will experience as he is decelerated by the rope. Typically, a 10g acceleration will cause a jet pilot to pass out due to all the blood being forced from the head to the legs. An acceleration of 16g's will cause damage to humans, i.e. the gravity load will be enough to break bones (your head weighs approximately 15 lbs, under 16 g's it will feel like it weighs 240 lbs).

UIAA limits:

UIAA defines a set of tests for measuring the performance of ropes. UIAA impact force test requires dynamic rope to be designed to limit the maximum dynamic load due to a falling weight of 80 kg (176 lbs) to 12 kN (2697 lb) when dropped 4.8 meters (15.7 ft) onto a 2.8 meter (9.2 ft) section of rope. The rope is passed over a 10 mm radius edge to simulate a carabiner. The test approximates a worst case fall (that's a fall factor 2, or falling twice the length of the rope). The impact force on the weight is limited on the first fall to 12 kN, and the rope must survive 4 falls. By limiting the impact force on the worst case fall, this test sets the design load that the rest of the climbing system must endure.

For purposes of comparison, here are the UIAA recommended minimum limits for strength in the safety system:[2]

Table 2: UIAA recommended Limits

Device	Minimum Limits
Anchors	25 kN (5620 lb)
Carabiners	20 kN (4496 lb)
Slings	22 kN (4945 lb)
Harnesses	15 kN (3372 lb)

By designing ropes to generate no more than the UIAA limit of 12 kN, the forces in that the different components of lead climbing protection should have an upper bound.

Fall Analysis.

In this section, we will illustrate how to compute the impact load factor for some different types of falls. In all the examples we will assume the belayer is using a dynamic rope and that the rope does not slip in the belay device.

Example: Fall Factor 0.

Let's assume that a climber falls while being topped roped with a dynamic rope and that the belayer has all the slack out of the rope. Figure 4 shows the geometry for such a fall. Notice that for this fall, *h* will be zero, and the impact load factor will be 2.0. At first glance, this does not seem correct. In order to understand why the impact fall factor should be 2, think about a weight at the top of a ladder with the rope just barely taut. If someone kicks the ladder from under the weight, then the weight will stretch the rope and bounce. The force at the peak stretch in the bounce will be a factor of two higher than the static force (i.e. when you are not bouncing).

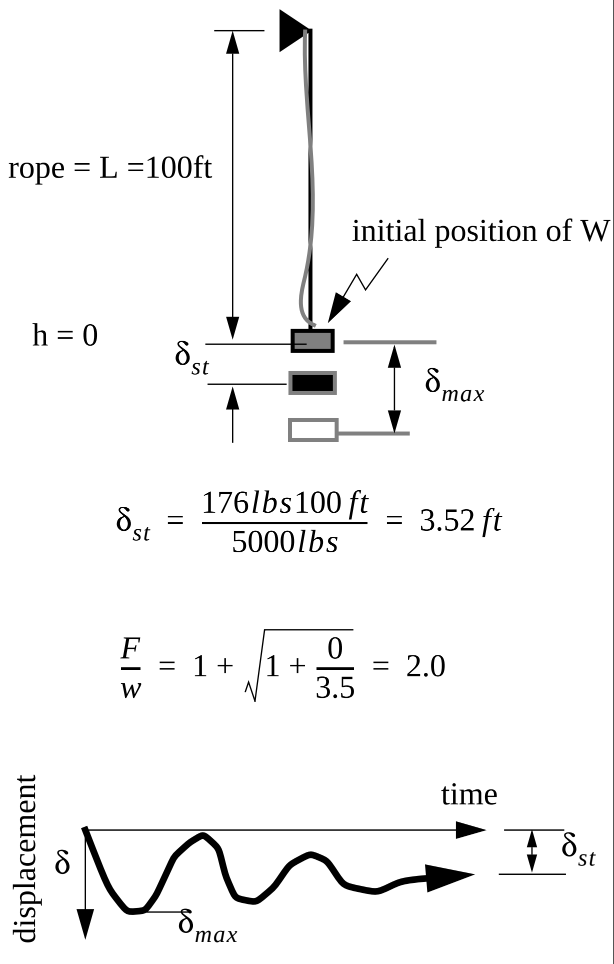


Figure 4**. Fall factor 0**. Height of fall, h = 0.0ft; Weight, W = 176 lbs, Static displacement = 3.5 ft. This gives an impact load factor of 2.0 or 352.0 lbs force in the rope.

For an actual system, it is easy to measure δ_{st}: simply hang on the rope. If δ_{st} is very small compared to h, then the dynamic force in the rope will be very large. Thus, you should be able to get a good idea of how much impact load-factor will be generated by bouncing on the rope. If you bounce your weight on the rope system, and you do not move very much, then it probably is not a good idea to fall.

Also shown in Figure 4 is a plot of what the displacement (force) would look like as a function of time. The displacement in the rope starts out at zero and climbs to its maximum. Once the peak displacement is reached, the climber will bounce about the static deflection until his motion is damped out by internal friction in the rope. Some ropes will have more damping than others. The peak force for a fall factor of zero will be the same regardless of the length of the rope. The duration of the force will increase as the rope length gets longer.

Example: Fall factor 1.

Now let's consider the case where a climber wants to test her rope. She goes to a nice high bridge and ties a 100 ft rope to the guardrail. After attaching the other end to her seat harness, she JUMPS off the bridge! Figure 5 show the geometry for this **fall factor of 1.**

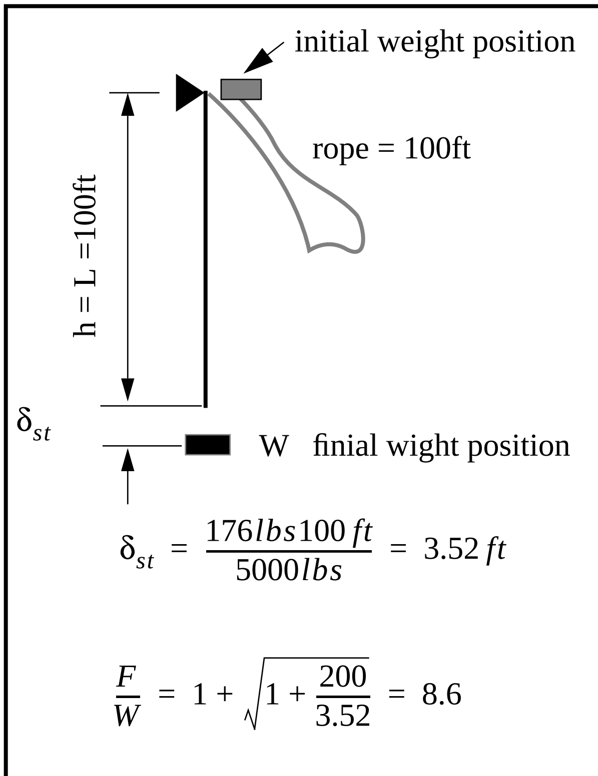


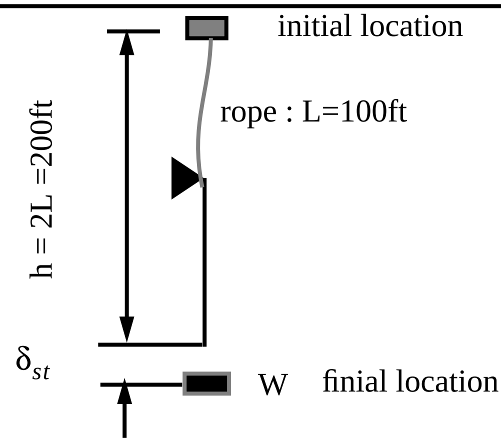
Figure 5**. Fall factor 1.** Height of fall, h = 100ft; Weight, w = 176 lbs, Static displacement = 3.52 ft. This give an impact load factor of 8.6 or 1514 lbs force in the rope.

In this case the rope and the height of the fall is the same. For the case shown, the impact load factor will be just over F/W=8.6 g's, or F = 1514 lb. If we assume static rope like PMI, then the modulus would be much higher and give F/W = 15.1 g's or F = 3035 lbs (don't try this at home! I know of two people that have taken 100+ ft. falls on PMI. In one case, the rope outer sheath was cut by their jumars, the sheath slid down the rope 1.5 feet, and the core melted and fused for 2 feet. The climber was uninjured and climbed back up the rope. I assume they did not reach the full 15 g's because of the energy absorbed by sheath cutting and slipping).

Example: fall factor 2.

Ok, now lets consider what has to be the worst case fall factor. A climber is the full rope length above the belayer as shown in Figure 6 The fall would be twice the length of the rope, or a **fall factor of 2**. Even though the fall is from twice the height, the impact load factor only increases to 11.4. Most people consider the fall factor of 2 to be the worst case impact load. It is important to note that a fall factor of 2 can be generated on a very short fall.

By measuring the dynamic load on a worst case fall, manufacturers can give their rope customers an idea of how stiff their ropes are compared to others. For rope comparison, manufacturers list the impact force for their rope assuming a fall factor of 1.78 with a weight of 80 kG (176 lb). Table 3 lists some typical Manufacturer's rope specifications.



$$\delta_{st} = \frac{176 lbs 100 ft}{5000 lbs} = 3.52 ft$$

$$\frac{F}{W} = 1 + \sqrt{1 + \frac{400}{3.52}} = 11.7$$

Figure 6**. Fall factor 2.** Height of fall, h = 200ft; Weight, w = 176 lbs, Static displacement = 3.52 ft. This gives an impact load factor of 11.7 or 2060 lbs force in the rope.

Example: The effect of rope length.

The example shown in Figure 7 shows two different scenarios. One that most sane climbers will avoid because of fear: a full rope length, fall factor 2 screamer. The other scenario is one that we all see on every climb that has a hanging belay: a 10 ft fall with only 5 feet of rope out. The

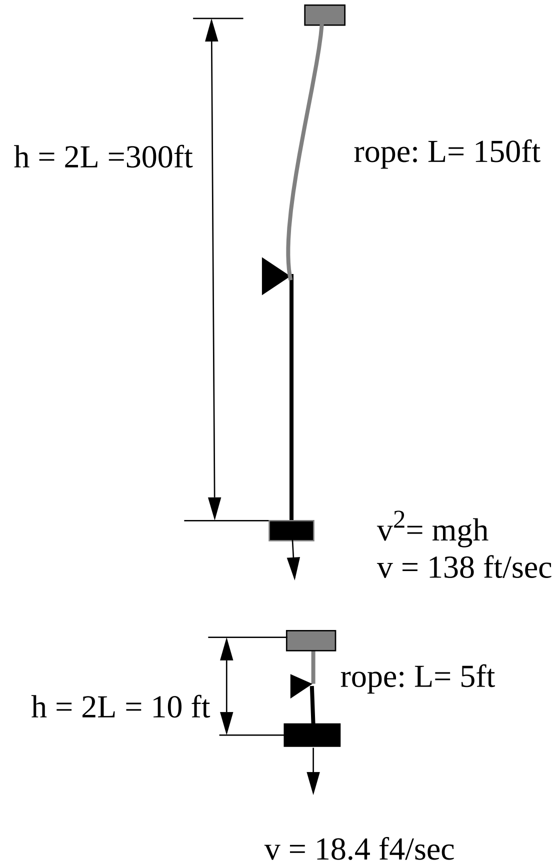


Figure 7. Q: Which rope system will generate the greater load in the rope? A: The load in the rope will be the same for both systems!

Table 3: Typical Rope Specifications[a] for UIAA fall test.

Diameter mm	Impact Forces kN (lbs)	Impact load factor F/W (g's)
9.8 (a)	10.7 (2400)	13.6
10.5 (a)	11.4 (2562)	**14.5**
10.8 (a)	10.2 (2292)	13.0
11.0 (a)	10.2 (2292)	13.0
10.0 (b)	9.8 (2203)	12.5
10.2 (b)	9.9 (2225)	12.6
10.5 (b)	9.4 (2113)	**12.0**
11.0 (a)	9.7 (	12.3

a. New England Rope, b. Mammut Rope. [3]

peak force for both of these falls will be the same. Even though the first fall is higher, there is more rope to absorb the fall energy.

The difference is that in the 300 ft fall, the duration of the force will be much longer than for the short fall. Even though the short fall's duration is much smaller, believe me, it will still yank very, very hard on your anchor and your body.

Leader Fall Analysis

Ok, so we conclude for the above analysis that the rope can catch a fall twice the length of the rope. Now, lets look in more detail at just what happens during a leader fall. Figure 8 shows a typical leader fall. The leader rope length (i.e. the "lead out") is L_2, above his last protection. The climber will fall a distance $h = 2L_2$. The static deflection is based on the total length of rope $L = L_1+L_2$. As an example consider L_1 = 10 ft. and L_2 = 20 ft. Assume a rope modulus of M=5000 lbs/ft./ft. and a W= 176 lb climber. This will give a load factor of F/W=10.0. The load in the rope would be F=1760 lb.

To see this in more detail, lets go back to Eq(14) and insert the rope stiffness directly from Eq(2). This gives

$$\frac{F}{W} = 1 + \sqrt{1 + \frac{2hM}{WL}} . \qquad \text{Eq(16)}$$

where L is the total length of the rope, $L_1 + L_2$, and h is the distance that the climber falls, $2L_2$.

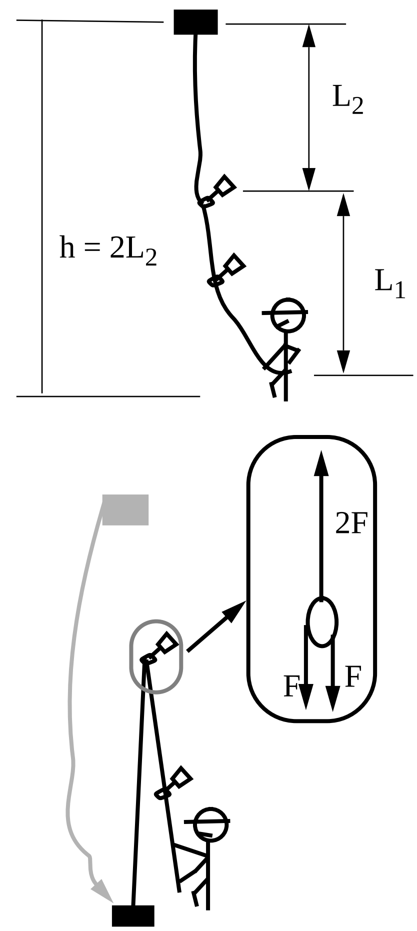
At the anchor point, the rope doubles back to form a 2:1 mechanical advantage. This generates twice the load at the anchor if we assume no friction. (The effects of friction will be considered later.)

Now, suppose that the climber is 100 ft. above his last anchor, L_2 = 100, then the total rope length will be L = 110 ft., to give a fall factor of 11.2. This would generate 1973 lbs of force in the rope and 3946 lbs of force on the anchor.

Typical cam anchors fail between 2000 - 3000 lb of static load, while chock type anchors on wires will also fail between 2000 - 3000 lbs. Hex style anchors on Spectra webbing typically are rated to around 5000 to 6000 lb. Bolted anchors in hard rock are typically good for 5000 to 6000 lb. So, you see, when we combine the 2:1 mechanical advantage with the impact load on the rope, we can easily generate enough force to pull out cam and wire type anchors. So, now for the question that every rock climber has asked at least once while on the rock.

How far can I lead out and still be safe?

Lets say you are 15 ft above a cam anchor with a rated strength of 2500 lbs and there is 50 ft of rope between that placement and the belayer. Just how safe are you? Figure 9 shows a plot that can be used to estimate the anchor forces generated for a given combination of lead outs.



$$\frac{F}{W} = 1 + \sqrt{1 + \frac{2hM}{W(L_1 + L_2)}}$$

Figure 8. Leader fall: Anchor load is twice the impact load factor.

To see how to use this graph, let's consider a couple of examples. Let's look at a climber that is 20 ft above his last anchor, L_2=20, and a total of 40 ft of rope out, or L_1 = 20. Thus, the climber will fall h=40ft with L=L_1+L_2= 40 ft. (fall factor 1). This places him at point A on the plot which corresponds to an anchor impact force of 3400 lbs. Since a cam anchor cannot support this much impact force, we hope that a good strong bolt is used for this anchor.

First, lets consider a bolted route. Typically, bolt manufactures claim that a well placed bolt can support 5600 lbs (25 kN). If we assume an old bolt in weak rock, such that the bolts can only support 3500 lb, then we can take a 20 ft lead beyond the last piece of pro at 20 feet. We can take an 80 ft lead from 80 feet. If we really have a bomb proof (nuclear, that is) anchor (5500 lbs), then we can take a 300+ fall and not fail the anchor. (Don't try this without lots of overhang: at 300 ft, the rock will be going past you at 95 m.p.h. You would not want to get out of your car if it was going this fast).

Now, lets look at cam anchors. Cams are typically strength rated for 2500 lb. We will assume that because of rock conditions, placement, etc. that a cam can support only 2000 lb of anchor force. Huber, 1995 has summarized some important findings on the strength of camming anchors and suggests that 2000 lbs may be overly optimistic.

Under these assumptions, a 176 lb climber would be limited to 7 ft of lead out above her pro at 40 ft, (point B), 15 ft. of lead out at 90 ft (point C), and 22 ft of lead out at 140 ft.

In general, anchors can be divided into two classes: high strength and low strength. The high strength class would include well placed bolts and well placed hexes on strong cord. The low strength class would include all active camming devices and nuts on wires. The difference between high strength anchors and low strength anchors needs to be emphasized. The failure to distinguish performance of the two allows some very bad assumptions on what is a safe lead out.

One problem is that many of the climbers are taking long leader falls on bolted routes, and then expecting their cams and wire chocks to hold similarly on lead climbs. A general rule of thumb for the low strength anchors is to not lead out more than 1/4 the length of the rope between the belayer and the highest piece of protection.

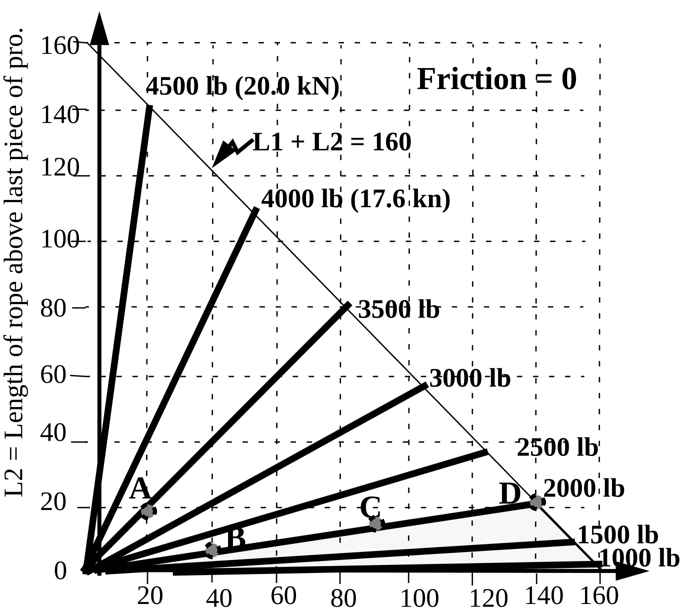


Figure 9Plot of impact load on anchor for W=176.0 lb, M = 7000 lb., for different combinations of lead out above last protection and distance from belayer to protection.

An interesting exercise is to work out the optimal number of anchors required for a 160 ft lead climb. First, we will assume that the first anchor is a great one, and that we can get off the ground 6 ft. A fall from just one foot above this anchor would fail a 2000 lb anchor!. Ok, so we put in a nice 4000 lb hex and climb to 12 ft and place a cam. At this point, we can climb about 2.5 ft above the 12 ft placement before the cam will overload at the 2000 lb limit. Now at 14.5 ft, we can climb another 3 ft to the next anchor, at 17.5 ft., 4 more ft to 21.5 ft, 5 ft to 26.5, 6 ft to 32.5, 7 ft to 39.5, 9 ft to 48.5, 12 ft to 60 ft., 14 ft to 75ft, 17 ft to 91 ft, 19 ft to 110 then 25 ft to 135, 30 ft to the top. That's a minimum of 14 placements. If, say, at 75 ft we lead out 20 ft instead of 17, we could unzipper all the pieces and hit the ground.

In contrast, if 3500 lb anchors are set, then we could (may not want to) set only 5 pieces at 10, 20, 40, and 80. (Oops, don't forget about the stretch in the rope.)

The effects of friction.

Any climber will tell you that the above analysis is nonsense because we did not include the effects of friction. Ok, lets redo the analysis and consider friction. Testing has shown that the friction on a rope that bends 180 degrees over a carabiner will reduce the load that the belayer feels by 52 percent (Soles, 1995). This friction can reduce the overall anchor load because the force in the belay side will not be as high. (see Figure 10) This effect is offset somewhat by the reduction in stretch of the rope as the climber falls.

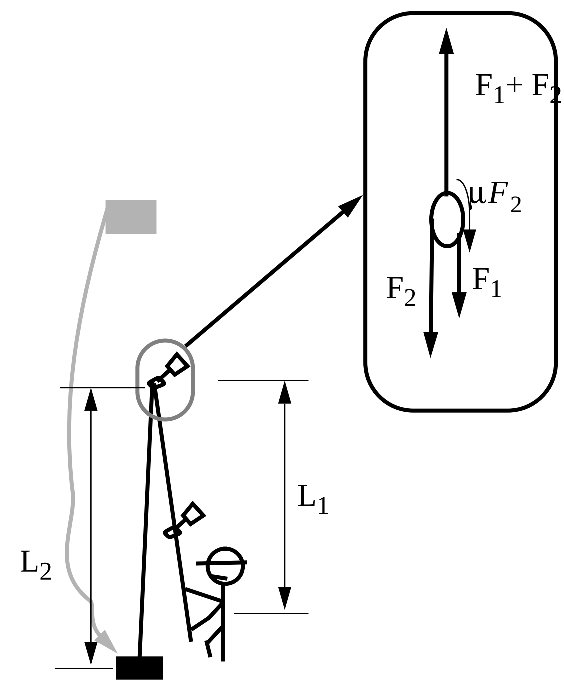


Figure 10Friction effects on a leader fall can reduce the effectiveness of the pulley effect at the anchor.

The friction force can be expressed as a fraction, μ, of the impact force. The impact force will be balanced by the friction force and the belay force:

$$F_2 = \mu F_2 + F_1 \qquad \text{Eq(17)}$$

or

$$F_1 = (1-\mu)F_2 \qquad \text{Eq(18)}$$

The total displacement of the load will be the sum of the displacements in the belay and climber side of the rope.

$$\delta = \delta_1 + \delta_2 \qquad \text{Eq(19)}$$

where

$$\delta_1 = \frac{F_1 L_1}{M} \text{ and } \delta_2 = \frac{F_2 L_2}{M}. \qquad \text{Eq(20)}$$

Substitution of δ_1 and δ_2 into Eq(18) gives:

$$\delta_1 = (1-\mu)\frac{L_1}{L_2}\delta_2 \qquad \text{Eq(21)}$$

The energy balance equation can still be used to compute the maximum displacement during a fall; however, we must include the work done by the friction force. The energy balance equation becomes:

$$PE = SE + W_f \qquad \text{Eq(22)}$$

where the work done by friction is equal to the frictional force integrated over the distance that the rope moves through the carabiner.

$$W_f = \int_0^{\delta_1} \mu F_2 dx \qquad \text{Eq(23)}$$

Using Eq(18) and Eq(20) we get,

$$W_f = \int_0^{\delta_1} \frac{\mu}{(1-u)}\frac{M}{L_1}x dx \qquad \text{Eq(24)}$$

or

$$W_f = \frac{M}{2L_1}\left(\frac{u}{(1-u)}\right)\delta_1^2 \qquad \text{Eq(25)}$$

Substituting Eq(21) into Eq(25) allows us to express the work done in terms of δ_2:

$$W_f = \frac{M}{2L_2}\mu(1-\mu)\frac{L_1}{L_2}\delta_2^2 \qquad \text{Eq(26)}$$

When $\mu = 0$ the work done by friction will be zero because the fractional force will be zero. When $\mu = 1$ the work done will also be zero because there will be no displacement through the carabiner.

The potential energy will be:

$$PE = mgh + mg(\delta_1 + \delta_2) \qquad \text{Eq(27)}$$

or expressed in terms of δ_2

$$PE = mgh + mg\left((1-\mu)\frac{L_1}{L_2} + 1\right)\delta_2 \qquad \text{Eq(28)}$$

The strain energy in the rope will be the some of the strain energy in the L_1 and L_2 sections:

$$SE = \frac{M}{2}\left(\frac{\delta_1^2}{L_1} + \frac{\delta_2^2}{L_2}\right) \qquad \text{Eq(29)}$$

or

$$SE = \frac{M}{2L_2}\left((1-\mu)^2\frac{L_2}{L_1} + 1\right)\delta_2^2 \qquad \text{Eq(30)}$$

Substitution of Eq(26), Eq(28), and Eq(30) into Eq(22) gives:

$$mgh + mg1 - \mu\frac{L_1}{L_2} + 1\delta_2 \qquad \text{Eq(31)}$$

$$-\frac{M}{2L_2}1 - \mu\frac{L_2}{L_1} + 1\delta_2^2 = 0$$

a quadratic equation in terms of the displacement of the length of rope above the last point of protection, L_2. After solving for the displacement, we can get the total force on the anchor as:

$$F_{anchor} = (2-\mu)\frac{M}{L_2}\delta_2 \qquad \text{Eq(32)}$$

Figure 11shows the solution to Eq(31) and Eq(32) for W= 176 lb, M = 7000 lb and u = 0.5. If μ = 0, then we get the same solution as shown in Figure 9.

The effects of friction are greatest when the lead out is long compared with the amount of rope out. To see this compare Figure 9 with Figure 11. For this case, high frictional forces at the carabiner reduces the mechanical advantage at the anchor.

Friction does not significantly change the 'safe' lead out for a 2000 anchor. For cases where the belay line is long compared to the lead out, the effects of friction at the anchor are offset by a stiffer overall response. To understand this, consider the case where the friction is perfect, $\mu = 1$, and we have no movement of the rope through the carabiner. This case would lead to a fall factor of 2 impact load. The good news is that the belayer will not feel as much force when the climber falls. The bad news is that the anchor will still feel about the same pullout force. Here we have considered the effects of friction on only one anchor. I am not sure how friction from a zig-zag rope system would effect the anchor loads.

Eq(33)

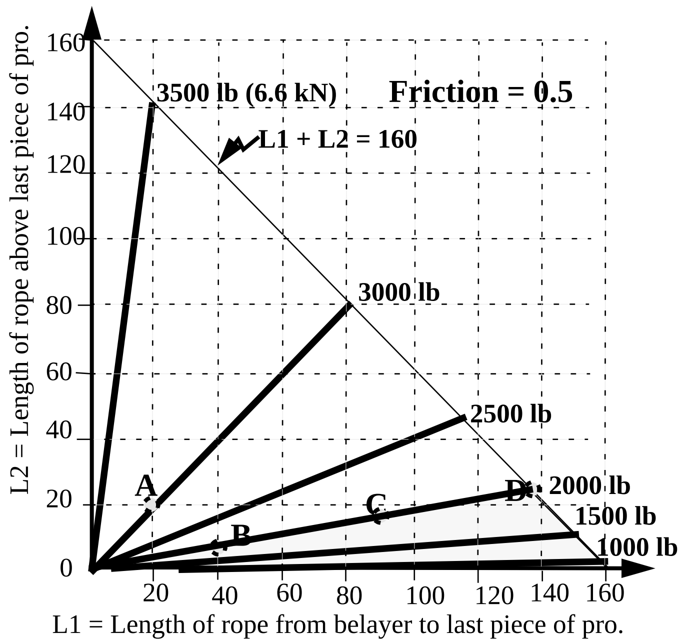


Figure 11. Plot of anchor load for W=176.0 lb, M = 7000 lb., friction = 0.5, for different combinations of lead out above last pro and distance to pro.

Modulus as a function of impact force.

Now that we have completed the analysis of a fall with friction, we can use it to model the UIAA test. This will allow us to compute the dynamic modulus of the rope from the test data generated during the manufacturer's qualification test. So, you ask, what's a dynamic modulus? Well..

Toomey, 1988, showed that a dynamic loading of nylon rope used for ocean towing can behave differently under dynamic and static conditions. Toomey showed that the dynamic modulus (the local secant modulus or the apparent modulus) can exceed the quasi-static stiffness by a factor of 3 or 4 depending on the rope construction. Figure 13 shows typical dynamic force deflection curves for marine rope.

In order to convert the impact forces that manufacturers supply with their ropes to an equivalent dynamic modulus, Eq(20) and Eq(31) will be applied to the UIAA standard test in such a way that the effective dynamic modulus is solved for as an unknown. In the UIAA drop test, a weight, W = 80 kG, is dropped h=4.6 meters with L_1 = 0.3 m and L_2 = 2.5 m. Figure 12 shows a plot of the solution of M in terms of the impact force that a climber feels. A dynamic rope that has a rated impact load of 10 kN would correspond to a rope modulus of 30.0 kN or 6744 lb. Here, we assumed a friction factor of $\mu = 0.5$; however, because the length of the L_1 side of the rope is so short, friction makes less than a 5% difference in the impact force in the rope.

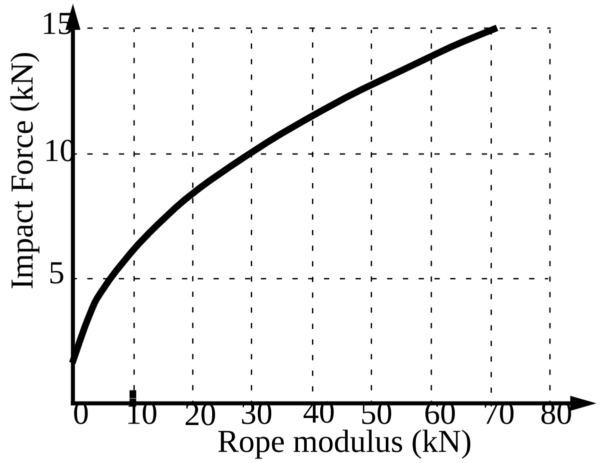


Figure 12. Impact force as a function of Rope modulus for UIAA test.

In the above calculations, we assumed that the rope modulus was constant, independent of stretch. In fact, the rope modulus is a function of rope stretch, which is evident from the lack of a straight line relationship between load and stretch in Figure 13 Assuming a linear relation for calculating maximum load should still be a good estimate of the strain energy in the rope, provided that the effective dynamic modulus is computed from the UIAA dynamic rope test data. I could not find any manufacturing data for dynamic climbing ropes that measures the rope modulus as a function of stretch.

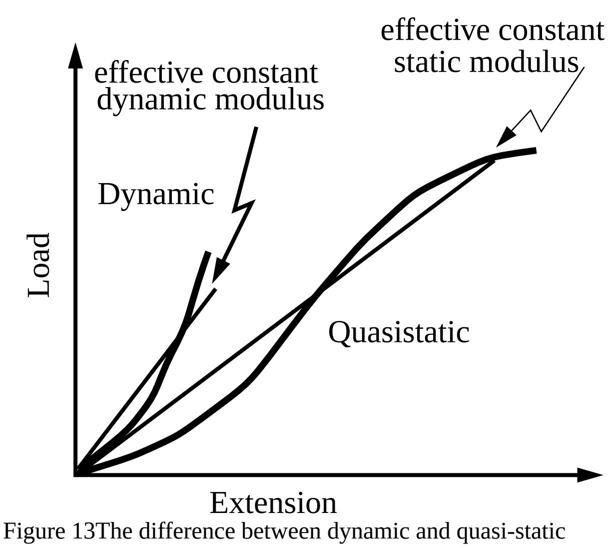


Figure 13The difference between dynamic and quasi-static moduli of marine nylon rope.(Toomey, 1988)

Ok, so now we can compute the effective dynamic modulus. However, everyone knows that an old rope does not feel and work like a new rope. Could the rope modulus can also change with rope use? When drop tests are performed on a rope, the measured modulus increases with each drop. The

UIAA test requires that the rope survive 5 drops. The impact force is determined from the first drop. It is not unusual for the impact force to increase by 30 to 60% after four drops. If the rope generated 10 kN of force on its first drop, and increased to 15 kN by its fourth drop, this would mean that the modulus must increase by almost a factor of 2.4! It's no wonder that the rope fails after a set number of falls. Basically, the rope becomes stiffer and stiffer with each fall to the point that you might as well be using a static rope.

But wait, that's not all! In addition to increasing with each impact load, the modulus can also increase from rappelling, jummaring, and lowering. Anything that subjects the rope to forces that can straighten the rope fibers can also increase the rope modulus.

Ropes used in climbing gyms are often subjected to many small falls, each of which tends to increase the rope modulus. C. Soles, 1995, performed tests on ropes used in a climbing gym for 2 weeks. Two of the ropes he tested broke on the first fall. Not good news.

As a rope is strained, the mechanical conditioning through the structural realignment and deformation of the fibers contribute to an increased stiffness of the rope. The outer sheath on a rope acts like a "Chinese finger trap" as it is tightened under load. By providing a constraining force, the outer sheath generates internal friction that must be exceeded to elongate the twisted core fibers. This frictional work will not be stored as strain energy and will be converted to heat as the rope is stretched. As a rope is cycled under load, a hysteresis effect will occur as the rope loads under one path and unloads under another. Toomey, 1988, measured the hysteresis for 1/2 inch Samson Ocean Towing rope, Figure 14.

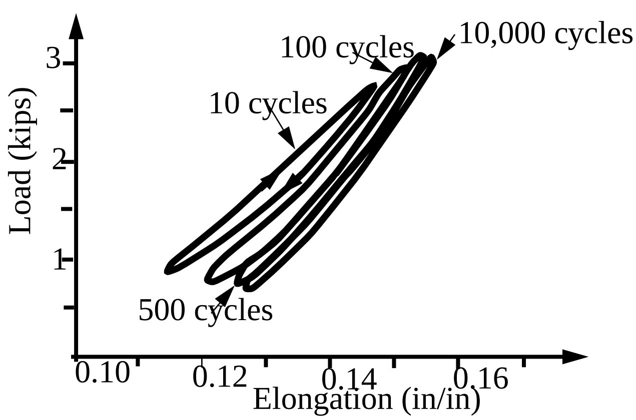


Figure 14. Hysteresis loops for nylon, dry, cycled at steady tension of 1.8 kips, frequency of 0.2 Hz and a strain amplitude of 0.017 in/in at the 10th, 100th, and 500th and 10,000th cycle. (from Toomey, 1988)

Two important observations can be made from Toomey's hysteresis measurement: 1). the dynamic modulus increased with load cycles; 2). the amount of hysteresis decreased with load cycles. **Does climbing rope experience the same sort of behavior? Can the rope be mechanically reconditioned to remove the effects of cyclic fatigue**?

Figure 15 shows typical results for the change in dynamic modulus for a rope as a function of cyclic loading. The good news is that the dynamic modulus (for this type of rope construction, anyway) approaches a constant as it is cycled. **How does the dynamic modulus for different dynamic climbing ropes behave under cyclic loading**?

Based on the behavior of ocean towing ropes, we would conclude that an old rope might be a tired rope. One sport climber that I talked to said that after each fall he "worked" the rope to recondition it. I do not know if a rope can be "reconditioned". Until I find out, I will have two ropes. The one in mint condition, that I will use for big wall climbs. After a fall on any rope, it will be used as an "old" rope for work in the rock gym, top-roping or for rappelling.

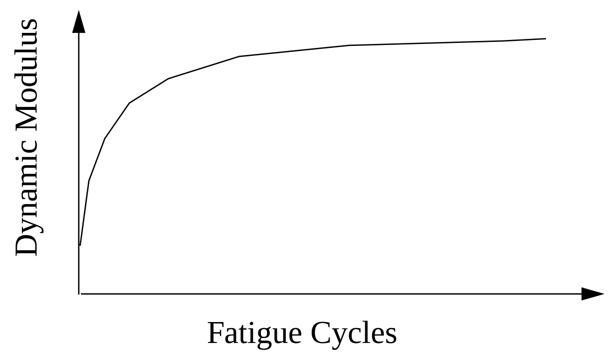


Figure 15. Typical change in dynamic modulus with fatigue cycles. [Pervorsek and Kwon, 1976]

Belay Devices: The good news!

In the above calculations for a leader fall, we have assumed that the belayer does not let the rope slip through his hands. Table 4 shows the typical breaking force of some belay devices.

Table 4: Breaking Forces for belay devices[a]

Type	Breaking Force (kN)
Figure 8	1.5
Stitch plates	2.0
ATCs	2.0
Munter Hitch	3.0
Grigri	9.0

a. Clyde Soles, Rock and Ice Magazine, Vol. 117, No. 68

Clearly, the dynamic breaking provided by a slipping belay device will limit the impact load. Caution should be used when using any of these belay devices, because the rapid slippage of the rope can burn the belayers hands. Also, anyone who has gone just a little too fast on rappell can attest to the fact that these belay devices require strict attention to prevent mishap.

The force limiting nature of dynamic belay devices clearly has ramifications for rescue belays. Two methods of rescue belay are common. One uses a set of double prusik knots to belay the load. The other method is more time consuming and consists of pulling the rope up through a belay device. Since static ropes are used for hauling systems, the use of a dynamic belay device could greatly reduce the impact loads should the system be shock loaded.

Deflection of a rope on a traverse.

Traverses often are used on climbing routes and are frequently used during rescue operations for safety belays. The impact loads that result when a fall occurs on a traverse can be quite high. The rope loads depends on the geometry of the traverse. Consider a climber crossing a traverse with only a locking carabiner clipped into the traverse. If the fall occurs at either end of the traverse, then the climber would slide toward the center of the traverse. As the climber slides toward the center of the traverse, friction will dissipate some of the energy. The worst case would be to assume that the climber is located such that when he falls, he will not slide. For a traverse with equal height anchors, this ‘horizontal equilibrium’ (i.e. no sliding) will be at the center of the rope. If the anchors are at different heights, then the ‘horizontal equilibrium’ will not be at the center, but at the point where the angles θ_1 and θ_2 shown in Figure 16 are equal.

As an example, we will look at the traverse system as shown in Figure 16 The bolts on the left were set 10 ft. above the bolts on the left, giving the traverse about a slope.

Before we can compute the dynamic response of this system, we must first compute the static deflection. (Remember that you can also measure the static deflection in the field.)

The equivalent stiffness of the traverse rope is computed by combining the stiffness of the rope to the left and the right of the weight. The actual formula for computing the equivalent stiffness is based on computing the stiffness along the length of the rope, then rotating the stiffness to the correct geometry. If you really want to see how this done see Martin[10].

The equivalent stiffness is

$$K_e = K_1(\sin\theta_1)^2 + K_2(\sin\theta_2)^2 \qquad \text{Eq(34)}$$

where $K_1 = \dfrac{M}{L_1}$ and $K_2 = \dfrac{M}{L_2}$ are the stiffness of the lengths of rope. θ_1 and θ_2 are the angles between the ropes.

The deflection given by the above formula would be:

$$\delta_{st} = \frac{W}{K_1(\sin\theta_1)^2 + K_2(\sin\theta_2)^2} \,. \qquad \text{Eq(35)}$$

Eq(34) and Eq(35) are only valid for cases were the angles do not change much due to the deflection. If the rope is straight to start with, then the angles will be zero, giving an infinite deflection. What really happens is that the rope deflects, and the resulting deflections make the problem geometrically non-linear. This does not mean that the problem cannot be solved, it just means that the math becomes to hard to do by hand, and a computer is recommended. There are several commercial computer programs that can compute this sort of non-linear deflection. As it turns out, you do not want to rig a traverse so taut that it has a near zero angle. If you have a near zero angle, then the loads on the anchor will be very high.

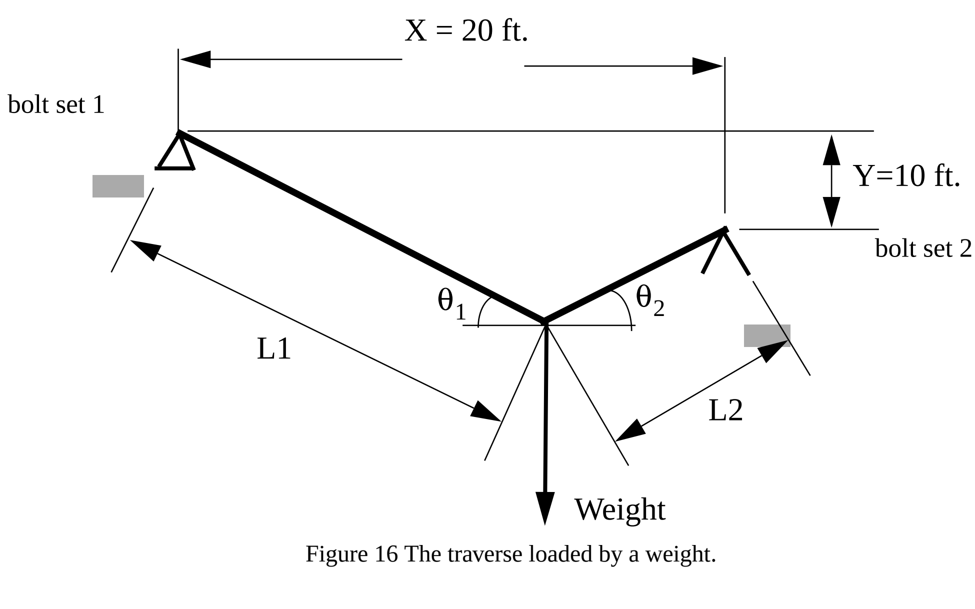


Figure 16 The traverse loaded by a weight.

Table 5: Loads on a traverse.

L (ft.)	h (ft.)	L1 (ft.)	L2 (ft.)	angle (deg)	F(lbs)
20	3	19.5	0.5	31.8	3635
21	3	19.0	1.98	35.95	1985
22	3	18.9	3.12	39.4	1645
23	3	18.9	4.07	42.34	1479
23	5	18.9	4.07	42.34	1862

Table 2 shows the dynamic impact forces computed for different lengths of PMI rope (M=20000 lbs/ft./ft.). The computations assume a 200 lb weight (160 lb with 40 lb pack). Notice how much difference 2.0 ft. of rope length make in the impact load factor!

Summary

We have presented equations for computing the dynamic impact load factor for typical rope systems used for rock climbing. The equations are based on the height of the fall, the deflection of the rope, and friction. Example calculations showed that it is easy to exceed the recommended maximum loads that are typical of cam type anchor devices. The calculations show that falls from any combination of lead out and belayed rope length usually will not exceed 4000 lbs of anchor force. However, falling from a leading out of more than 1/4 the belayed rope length could generate more than 2000 lbs of anchor force, the approximate force need to pull-out some types of climbing protection.

Recommendations:

- Build your anchors to withstand 25 kN (5500 lb) when possible.
- If you are going to use wire chocks and cams that have a typical strength of 2000 lbs, then don’t lead out more than 1/4 the belayed rope length.
- Never exceed your climbing abilities on a big wall climb. Test your skills at a rock gym, on bolted routes, or under top-roped conditions.
- Use only ‘new’ ropes for lead climbing. To protect against shock loading of the anchors, use a rope with a low modulus or impact force rating.
- Use a dynamic belay device.

Future Work

In order to design safer climbing anchors it is necessary to understand the forces generated from climbing ropes. While the above methods present a first order prediction of the forces involved in a climbing rope, some measurements that would be useful in better predicting forces on anchors are:

- Static force-deflection curve.
- Dynamic force-deflection curve.
- Dynamic force-deflection curve for ropes subjected to different cyclic loadings such as repeated rappelling, climbing, or short falls.
- Measure the dynamic modulus and hystereses of ropes after they are subjected to repeated falls.
- Determine the strain-rate dependence of climbing rope. (Dynamic force-deflection at different strain rates.)
- Preform dynamic tests that simulate a leader fall and measure the force.vs. time at the anchor and the belayer.

Dedication

This work is dedicated to Dr. Carlos Abad, Ms. Jane Teenessen, and Dr. Glen Tietjen who died from a fall of 817 feet on June 23, 1996.

References

1. Bill Klimack, “Strength of Climbing Systems, Nylon Highway, No 31, National Speleological Society.
2. Estimated from: “Understanding Shock Load”, http://www.infosphere.com/petzl.
3. http://www.ewalker.com/adgear/dr.htm
4. Spotts, M. F. 1978, Design of Machine Elements 5th ed.
5. C. J. Toomey, 1988, “The Dynamic Behavior of Nylon and Polyester Rope Under Simulated Towing Conditions,” Masters Thesis, MIT.
6. Pervorsek, D.C. and Kwon, Y. D., 1976, “Structural Interpretation of the Endurance of Polymeric Fibers,” J. Macromol., Sci-Phys, B12940, p. 447-485.
7. Wu, Hwai-Chung, 1992, “An Energy Approach for Rope-strength Prediction,” Journal of the Textile Institute, Vol. 83, p 542-549.
8. Huber, Nicholas J., “An Analysis of Active Anchors Used by Lead Climbers,” Department of Mechanical Engineering, Univ. of Wisconsin, World Wide Web, Undergraduate Eng. Review, 1995.
9. Soles, C., “Single-Rope Buyers Guide, Every thing you always wanted to know about ropes but didn’t know to ask,” Rock and Ice Magazine, Vol. 117, No. 68.
10. Introduction to matrix methods of structural analysis, H. Martin, 1966.